NAG C Library Function Document

nag_quartic_roots (c02alc)

1 Purpose

nag_quartic_roots (c02alc) determines the roots of a quartic equation with real coefficients.

2 Specification

```
void nag_quartic_roots (double e, double a, double b, double c, double d,
          double zeror[], double zeroi[], double errest[], NagError *fail)
```

3 Description

nag_quartic_roots attempts to find the roots of the quartic equation

$$ez^4 + az^3 + bz^2 + cz + d = 0,$$

where e, a, b, c and d are real coefficients with $e \neq 0$. The roots are located by finding the eigenvalues of the associated 4 by 4 (upper Hessenberg) companion matrix H given by

$$H = \begin{pmatrix} 0 & 0 & 0 & -d/e \\ 1 & 0 & 0 & -c/e \\ 0 & 1 & 0 & -b/e \\ 0 & 0 & 1 & -a/e \end{pmatrix}.$$

Further details can be found in Section 6.

To obtain the roots of a cubic equation, nag_cubic_roots (c02akc) can be used.

4 Parameters

1: **e** – double *Input*

On entry: e, the coefficient of z^4.

Constraint: **e** $\neq 0.0$.

2: **a** – double *Input*

On entry: a, the coefficient of z^3.

3: **b** – double *Input*

On entry: b, the coefficient of z^2.

4: **c** – double *Input*

On entry: c, the coefficient of z.

5: **d** – double *Input*

On entry: d, the constant coefficient.

6: **zeror[4]** – double *Output*

7: **zeroi[4]** – double *Output*

On exit: **zeror**$[i-1]$ and **zeroi**$[i-1]$ contain the real and imaginary parts, respectively, of the ith root.

8: **errest[4]** – double *Output*

On exit: **errest**$[i-1]$ contains an approximate error estimate for the ith root.

9: **fail** – NagError * *Input/Output*

The NAG error parameter (see the Essential Introduction).

5 Error Indicators and Warnings

NE_REAL

On entry, **e** $= 0.0$.
Constraint: **e** $\neq 0.0$.

NE_C02_OVERFLOW

The companion matrix H cannot be formed without overflow.

NE_C02_NOT_CONV

The iterative procedure used to determine the eigenvalues has failed to converge.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

6 Further Comments

The method used by the routine consists of the following steps, which are performed by routines from LAPACK.

(a) Form matrix H.

(b) Apply a diagonal similarity transformation to H (to give H').

(c) Calculate the eigenvalues and Schur factorization of H'.

(d) Calculate the left and right eigenvectors of H'.

(e) Estimate reciprocal condition numbers for all the eigenvalues of H'.

(f) Calculate approximate error estimates for all the eigenvalues of H' (using the 1-norm).

6.1 Accuracy

If **fail.code** $=$ **NE_NOERROR** on exit, then the ith computed root should have approximately $|\log_{10}(\textbf{errest}[i-1])|$ correct significant digits.

6.2 References

Golub G H and Van Loan C F (1996) *Matrix Computations* Johns Hopkins University Press, Baltimore (3rd Edition)

7 See Also

nag_cubic_roots (c02akc)

8 Example

To find the roots of the quartic equation

$$z^4 + 2z^3 + 6z^2 - 8z - 40 = 0.$$

8.1 Program Text

```
/* nag_quartic_roots (c02alc) Example Program.
 *
 * Copyright 2000 Numerical Algorithms Group.
 *
 * NAG C Library
 *
 * Mark 6, 2000.
 */

#include <nag.h>
#include <nag_stdlib.h>
#include <nagc02.h>

int main(void)
{
  double a, b, c, d, e;
  double *errest=0, *zeroi=0, *zeror=0;
  Integer i;
  Integer exit_status=0;
  NagError fail;

  INIT_FAIL(fail);
  Vprintf("c02alc Example Program Results\n\n");
  if
  (
    ! (errest = NAG_ALLOC(4,double)) ||
    ! (zeroi  = NAG_ALLOC(4,double)) ||
    ! (zeror  = NAG_ALLOC(4,double))
  )
    {
      Vprintf ("Allocation failure\n");
      exit_status=-1;
      goto END;
    }

  /*  Skip heading in data file */
  Vscanf("%*[^\n]");
  Vscanf("%lf %lf %lf %lf %lf", &e, &a, &b, &c, &d);

  c02alc  (e, a, b, c, d, zeror, zeroi, errest, &fail);
  if (fail.code == NE_NOERROR)
    {
      Vprintf(" Roots of quartic equation          Error estimates\n");
      Vprintf("                                   (machine-dependent)\n\n");
      for (i = 0; i <= 3; ++i)
        {
          Vprintf("%s %10.5f %10.5f%s        %g\n", " z =",
                  zeror[i], zeroi[i], "*i", errest[i]);
        }
    }
```

```
  else
    {
      Vprintf("Error from c02alc.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
 END:
  if (errest) NAG_FREE (errest);
  if (zeroi)  NAG_FREE (zeroi);
  if (zeror)  NAG_FREE (zeror);
  return exit_status;
}
```

8.2 Program Data

```
c02alc Example Program Data
 1.0   2.0   6.0  -8.0  -40.0 : Values of e, a, b, c and d
```

8.3 Program Results

```
c02alc Example Program Results

 Roots of quartic equation          Error estimates
                                   (machine-dependent)

 z =    2.00000    0.00000*i        3.38974e-15
 z =   -2.00000    0.00000*i        5.29396e-15
 z =   -1.00000    3.00000*i        4.54379e-15
 z =   -1.00000   -3.00000*i        4.54379e-15
```